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Energy spectrum of alpha particles emitted in fission

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The motion of the alpha particles occasionally emitted in fission is studied. The effects of the shape and orientation of the fission fragments on this motion are considered. It is shown that, while the final kinetic energy of the alpha particle is not sensitive to the shape of the fission fragments, it is greatly influenced by the orientation of the fragments. It is shown also that agreement between the calculated and the observed spectrum of the final kinetic energy of the alpha particle is possible even when the alpha particle is assumed to be emitted with little or no kinetic energy. This result is important because it is widely believed that experimental data are consistent only with high initial kinetic energy of the alpha particle.

1 INTRODUCTION

The motion of the long-range alpha particles occasionally emitted in fission has been, and continues to be, studied extensively. Interest in this subject stems from the fact that much remains to be learnt about the configuration of fissioning nuclei, and from the belief that the motion of the alpha particles emitted in fission contains important clues to that configuration. The initial conditions of the motion of the alpha particles are determined by the configuration of the fissioning nuclei. From the initial conditions consistent with the observed motion of the alpha particles, therefore, one may infer possible configurations of fissioning nuclei. Thus the problem is essentially an inverted one of determining, not the motion ensuing from a known set of initial conditions but the initial conditions from the observed motion.

The imprint of the initial conditions upon the motion of the alpha particles is sought from two features: the *angular* and *energy* distribution of the alpha particles. These features may be summarized as follows. First, the angular distribution of the alpha particles has been observed, by Fraenkel (1967), Raisbeck & Thomas (1968), Boneh and co-workers (1967) and Rajagopalan & Thomas (1972), to be peaked at an angle (made by the trajectory of the alpha particle with that of either of the major fission fragments) of about 90° and to have a full-width-at-half-maximum (FWHM) of 20–30°. Second, the energy distribution of the alpha particles has been observed to be peaked at about 15 MeV and to have a FWHM of about 13 MeV. From these two features many inferences have been made of the initial parameters appropriate for a fissioning nucleus. There has been general agreement among these inferences But there has been disagreement, too. By far the most serious dispute has been about the *initial* kinetic energy with which the alpha particle exists following scission Opinions on this question may be divided into two categories; there are those, e.g., Ertel (1969), Fong (1969) and Vitta (1971), who believe that the alpha particles (and fission fragments) are produced with little initial kinetic energy, and there are those, e.g., Boneh and co-workers (1967), Rajagopalan & Thomas (1972), and Nix & Swiatecki (1965), who believe that they are produced with considerably high kinetic energy

2 The Effects of the Shapes and Orientation of Fragments

In this paper, we present results of computer calculations performed with the intention of shedding more light on, especially, the disputed initial kinetic energy of the alpha particles emitted in fission The calculations take into account the *size*, *shape* and *orientation* of the fission fragments. This represents a point of departure from most of the previous calculations in which the fission fragments have been regarded to be *points*, with mass and charge but without size, shapes or orientation in space The calculations reported in this paper were performed first by determining the electrostatic field due to the two major fission fragments as a function of the shape and orientation of each fission fragment, and then by following the motion of the alpha particle in that field

For simplicity each of the two major fission fragments was assumed to be an ellipsoid, with the major and semi-major axes a and b ($i = 1, 2$) as shown in figure 1 The electrostatic field $\boldsymbol{E}$ seen by the alpha particle at the position r is the superposition of the electrostatic fields of the two major fission fragments, and is of the following form.

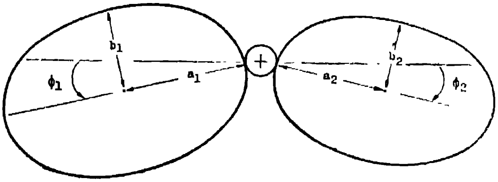

Fig. 1. A schematic diagram showing the relative positions, shapes and orientation of the fission fragments immediately after scission. The alpha particle exists from the neck of the fissioning nucleus

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{i=1}^{2} \left[\frac{q_i(\boldsymbol{r}-\boldsymbol{r}_i)}{4\pi\epsilon_0 |\boldsymbol{r}-\boldsymbol{r}_i|^3} - \frac{Q_i}{4\pi\epsilon_0} \nabla \frac{(3\cos\theta_i - 1)}{|\boldsymbol{r}-\boldsymbol{r}_i|^3} \right] \tag{1}$$

where $\boldsymbol{r}_i$, q_i and $Q_i = 2q_i(a_i^2 - b_i^2)/5$ are the position, charge and electric quadrupole moment of the i-th fission fragment, respectively. θ_i is the angle made by $(r - r_i)$ with the axis of the i-th fission fragment. In eq (1), the second term on the right-hand side represents the dependence of the electrostatic field seen by the alpha particle upon the shapes of the fission fragments. The effect of this dependence of the electrostatic field on the shapes of the fission fragments upon the motion of the alpha particle has been studied by following the motion of the alpha particle in the field $\boldsymbol{E}$. Save for the inclusion of the quadrupole terms in eq. (1), the computer calculations were performed in much the same way as has been described in detail previously.

We have also studied the effect of the orientation of the fission fragments at scission. This was done by assuming that at scission the axis of the i-th fission fragment makes an angle ϕ_i with a reference line drawn through the initial position of the alpha particle (figure 1) For ease in computation, but without any loss of generality, the reference line was chosen so that the fission fragments made equal angles with it—that is, $\phi_1 = \phi_2 = \phi$ The dependence of the electrostatic field $\boldsymbol{E}_i$ of the i-th fission fragment on the orientation of the fragment may be obtained as follows. First, the fission fragment is placed with its centre at the origin and oriented along the z-axis of the coordinate system relative to which $\boldsymbol{E}_i$ is to be given The coordinate system (with it the fission fragment) is then rotated by angle ϕ_i, counterclockwise for $i = 1$ and clockwise for $i = 2$ (figure 1). In the rotated coordinate system the electrostatic field of the fission fragment has terms identical to those in eq (1) The desired result is obtained by transforming this field back to the original (unrotated) coordinate system. But before the electrostatic fields of the fission fragments are sueperposed to give the total field seen by the alpha particle, it is necessary to translate the fission fragments through suitable displacements d_i so that their centres lie at their correct initial positions, and once more to transform the electrostatic fields of the fission fragments accordingly. In this way, one obtains the electrostatic field seen by the alpha particle for various values of ϕ_i. This process is straightforward (if tedious) Here there is need to write the final result only symbolically as follows,

$$\boldsymbol{E}(\boldsymbol{r}) = \sum_{i=1}^{2} [T(d_i)R(\phi_i)]\boldsymbol{E}'_i(\boldsymbol{r}'), \qquad \ldots \quad (2)$$

where T and R represent translation and rotation by d_i and ϕ_i, respectively

3. Results

The initial parameters needed in computations of the kind-reported here are the initial position $\boldsymbol{r}_0$ (relative to the major fission fragments from which the alpha particle exists, the instant t_0 (following scission) at which this happens, the initial kinetic energy E_0 of the alpha particles and the initial direction of flight of the alpha particle (which may be represented by the angle θ_0 between the initial velocity of the alpha particle and the line joining the centres of the major fission fragments). The influence of these initial parameters upon the distributions of the final kinetic energy E and the final direction of flight θ of the alpha particles has been studied before. In this work we have placed emphasis on especially two other initial parameters : the initial shapes and orientation of the fission fragments The calculations were performed using data for Cf^{252} Only the most probable fission mode, in which the masses m and charges q of the fission fragments are in the ratio $m_1/m_2 = q_1/q_2 = 1 \cdot 4$, was considered. The initial positions of the fission fragments were taken to be those shown in figure 1 The numerical values of the major and semi-major axes a_i and b_i were obtained from the liquid-drop formula for the nuclear radius :

$$r_0{}^2 A_i{}^{2/3} = \tfrac{1}{2}(a_i{}^2+b_i{}^2), \; r_0 = 1 \cdot 2\times 10^{-15}\mathrm{m}, \qquad \ldots \quad (3)$$

where A_i is the mass number of the i-th fission fragment. The initial positions of the fission fragments depend on the shapes of the fragments, which may be represented by the parameter S_i defined as follows.

$$S_i = \frac{a_i{}^2-b_i{}^2}{a_i{}^2+b_i{}^2}. \qquad \ldots \quad (4)$$

The initial positions of the fission fragments, of course, also depend on the initial orientation ϕ_i of the fragments. From assumed sets of S_i and ϕ_i, one may work out the corresponding initial positions of the fission fragments.

We have studied the effects of the shapes S_i and orientations ϕ_i on the distribution of the final direction of flight θ of the alpha particle, concluding that the distribution of θ is not sensitive to variations in S_i and ϕ_i However, our main objective in this study was to determine the effect of S_i and ϕ_i on the distribution of the final kinetic energy E of the alpha particle Our calculations have shown that the shapes S_i of the fission fragments have little effect on the final kinetic energy of the alpha particle (figure 2). On the other hand, the orientation of the fission fragments has great influence on this energy. Curves (a), (b) and (c) in figure 2 show the (calculated) dependence of the final kinetic energy of the alpha particle on the shapes of the fission fragments for three different orientations of the fragments Curve (a) of figure 2 shows that an alpha particle which merely "breaks" off from the fission fragments (that is, $E_0 = 0$) at a point on the line joining the centres of the fragments (that is, $\phi_1 = \phi_2 = 0$) cannot attain the observed average final kinetic energy of 15 MeV. But curves (b) and (c) of figure 2 show that it can if it breaks off from fission fragments which are oriented in the form of a "V" so as to make angles ϕ_1 and ϕ_2 with a reference line drawn through the scission point (figure 1) Nor do the angles ϕ_1 and ϕ_2 have to be large. Figure 2 shows that ϕ_1 and ϕ_2 may each be only 0·1 radian (that is, about 6°), or less.

The dependence of the final kinetic energy of the alpha particle on the orientation of the fission fragments is depicted in figure 3 for various shapes of

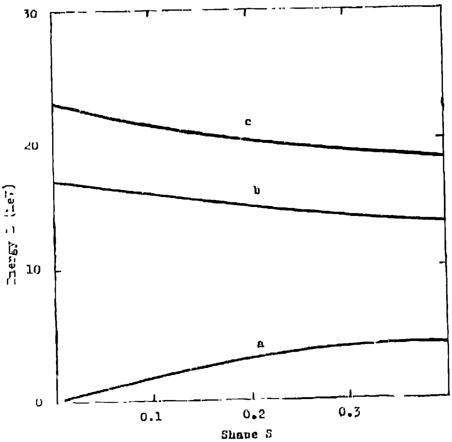

Fig 2 The effect of the shape S of the fission fragments on the final kinetic energy E of the alpha particle when the initial fragment orientation ϕ is 0 radian [curve (a)] 0·1 radian [curve (b)] and 0·2 radian [curve (c)]

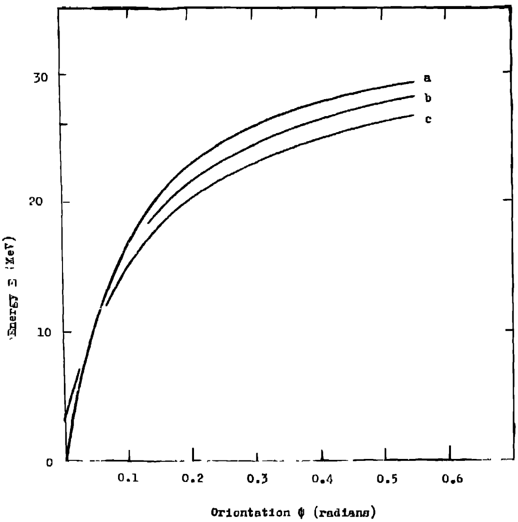

Fig 3 The effect of the fragment orientation ϕ on the final kinetic energy E of the alpha particle when the shape S of the fragments is 0 [curve (a)] 0·1 [curve (b)] and 0 2 [curve (c)].

fragments. Once more we see only a weak influence of the shape of the fission fragments on the final kinetic energy of the alpha particle and a very strong influence on this energy by the orientation of the fragments Figure 3 may be used to estimate the effect of a distribution of the orientation ϕ of the fission fragments on the distribution of the final kinetic energy E of the alpha particle We have estimated that a spread in the orientation of the fragments of 2^0, or less, about $\phi - 3^0$ gives rise to a spectrum of the final kinetic energy E of the alpha particle consistent with what has been observed This estimate seems to us to be both possible and plausible.

4. Conclusion

Several authors, e g. Boneh co-workers (1967), and Rajagopalan & Thomas (1972) have found it necessary to assume that the alpha particle exists with high-initial kinetic energy. From this they have concluded that the statistical model of nuclear fission, which predicts low initial kinetic energy, does not provide a correct picture of the configuration of fissioning nuclei Our calculations, which have been performed assuming little or no initial kinetic energy, have shown, however, that agreement with experiment is possible even at low initial kinetic energies This, of course, does not prove that the statistical theory provides a correct picture of fissioning nuclei. But it indicates that the theory cannot be impugned merely because it predicts low initial kinetic energies of the fission fragments

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